

Giant positive magnetoresistance and localization in bosonic insulators

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We study the strong localization of bosons, motivated by several experiments that suggest a bosonic superconductor-insulator transition in strongly disordered films. In the insulator, unlike for fermions, nearly all scattering paths between low-energy sites contribute positive amplitudes, which interfere constructively in the hopping matrix element. The localization length of bosonic excitations shrinks as the constructive interference is suppressed by a magnetic field. This entails a giant positive magnetoresistance, opposite to the analogous effect in strongly localized fermions. Within the forward scattering approximation, the localization length is found to increase with decreasing energy. Applying the same technique to hard core bosons on a Bethe lattice with large connectivity, we find the superfluid to emerge without concomitant closing of a mobility gap in the insulator.

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Many disordered insulators display some form of variable range hopping transport [1], which reflects the localization of the electrons close to the Fermi level [2]. When the hopping lengths are much larger than the distance between impurities, transport is very sensitive to orbital magnetic fields. The latter suppresses the destructive interference among alternative paths of electrons, and thus weakens the localization, leading to a strong negative magnetoresistance (MR) [3–5]. This is a non-trivial manifestation of quantum interference in disordered insulators. In this Letter we establish the bosonic counterpart of the effect, which turns out to have the opposite sign. The MR in the hopping regime thus serve as an indicator for the quantum statistics of the conducting carriers.

Recent experiments in strongly disordered, superconducting InO_x films [6, 7] have shown that a magnetic field not only destroys rapidly the already weak superconductivity [8], but also induces a giant MR peak in the ensuing insulating state, while at largest fields the resistance often drops back close to its normal state value. Similar peaks in MR have been reported in amorphous films of TiN [9], Bi [10], and in patterned films [11], as reviewed in [12].

The giant positive MR in the vicinity of the superconducting transition is intriguing. Mechanisms such as shrinking wavefunctions or spin blocking effects of weakly interacting electrons, which may play a role in semiconductors [1], hardly apply to these systems [13]. Instead, many experimental observations in transport [6–9, 11, 14, 15] and tunneling microscopy [16, 17], as well as theoretical model studies [18–20] suggest the crucial importance of remnant electron pairing in the insulator, despite the absence of global phase coherence [21]. In the presence of strong disorder, the resulting insulator is expected to be a Bose glass [22], whose low energy excitations are localized by disorder, but do not exhibit a spectral gap. The transport properties of such complex interacting systems pose interesting conceptual questions,

which are not fully resolved yet [13, 23, 24].

While it is natural to expect a magnetic field to increase the resistance of a Cooper pair insulator (in continuation of its detrimental effect on phase coherence in the superconductor), there is no satisfactory microscopic explanation of the giant effects seen in experiments to date. Ref. [25] attributed the MR peak to the survival of superconducting islands, whose properties are tuned by the field. A Coulomb blockade was postulated to occur in the islands, hampering transport of electrons, but not of pairs. Beloborodov et al. [26] considered an array of normal and superconducting grains. They showed that the magnetic field tunes the gaps of pair and electron excitations, resp., with opposite trends, which predicts a peak in MR. Ref. [27] interpreted the peak as reflecting the first oscillation of the charging energy of a clean array of Josephson islands in a magnetic field. However, the models [26, 27] hardly do justice to the complexity of the experimental systems [6] in which no well defined granular structure exists, and the spectral gap for pairs should be washed out by strong disorder. [18] The latter suggests that it is rather pair localization due to disorder, which induces the insulating behavior. [13]

In this Letter we study localization, MR and mesoscopic interference effects in *bosonic* insulators. These are also relevant for recent experiments with interacting cold atoms in strong disorder potentials and artificial gauge fields. [28]

We consider a simple model of bosonic or fermionic insulators: a lattice of sites, which can accommodate only one quantum particle, assuming strong onsite repulsion. This is the standard Anderson model for fermions [2, 3]. For hard core bosons the model was introduced by Ma and Lee [29] who considered disordered superfluids in terms of preformed pairs (pseudospins). The calculations below can easily be generalized to grains or islands hosting many particles, as long as the charge gap on typical grains is much bigger than the hopping amplitude be-

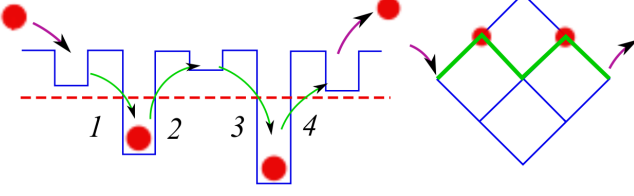


FIG. 1: In the configuration on the right, many paths contribute to the Green's function G_{ij}^R between the low energy sites i, j . The locator expansion yields the matrix element in 4th order perturbation theory as a sum over 6 paths. The sign of fermion amplitudes depends on the number of occupied sites on each path (shown on the left) whereas the paths for bosonic low energy excitations always have positive amplitudes. A magnetic field suppresses that constructive interference, while fermions display the opposite effect.

tween grains. A similar model was studied in Ref. [30], but the importance of quantum statistics and many body effects on localization and interference was missed.

We consider sites i with a random onsite energy ε_i , uniformly distributed in $\varepsilon_i \in [-W, W]$. They are weakly coupled by a tunneling amplitude $t_{ij} = t$ between nearest neighbors,

$$H = \sum_i \varepsilon_i n_i - \sum_{\langle i,j \rangle} t_{ij} (b_j^\dagger b_i + b_i^\dagger b_j), \quad n_i = b_i^\dagger b_i. \quad (1)$$

b_i^\dagger, b_i are creation and annihilation operators of fermions or hard core bosons, resp. They satisfy the commutation relations $[b_i, b_j]_B = 0$, $[b_i^\dagger, b_j]_B = \delta_{ij}[1 - 2B(1 - n_i)]$, where $[\cdot, \cdot]_B$ is the commutator or the anticommutator for bosons ($B = 1$) or fermions ($B = 0$), resp. In the presence of a magnetic field, the hopping acquires a phase $t_{ij} = te^{-i\phi_{ij}}$, the sum of phases ϕ_{ij} around a plaquette being proportional to the flux threading it.

We focus on the strongly insulating regime $t \ll W$, where hopping transport is expected at low temperatures. A key element characterizing disordered insulators is the localization length, ξ . For non-interacting fermions it is well-defined as the (log-averaged) inverse spatial decay rate of single particle wavefunction amplitudes. In contrast, hard core bosons are inherently interacting, requiring a generalization of this concept. In the limit $t = 0$ single particle excitations correspond to the addition or removal of a particle on given sites. For small hopping $t/W \ll 1$, these excitations adiabatically deform into dressed quasiparticle excitations, which are still well localized in space. In fact, one expects that *all* low energy excitations remain discrete and localized in this limit [13, 23, 24]. The spatial properties of such quasiparticle excitations are well captured by the retarded Green's function ($A(t) = e^{iHt} A(0) e^{-iHt}$)

$$G_{i,0}^R(t - t') = -i\Theta(t - t') \langle [b_i(t), b_0^\dagger(t')]_B \rangle. \quad (2)$$

It describes the amplitude of finding an extra particle at site i , and at time t after adding a particle on site 0. As in early studies of the Hubbard model [31], it is useful to study the equation of motion

$$i \frac{d}{dt} G_{i,0}^R(t - t') = \delta(t - t') \delta_{i,0} \langle [b_0(0), b_0^\dagger(0)]_B \rangle - i\Theta(t - t') \langle [\dot{b}_i(t), b_0^\dagger(t')]_B \rangle. \quad (3)$$

This is the starting point for a locator expansion in powers of the hopping t/W [2]. It is easy to show that

$$i\dot{b}_i(t) = [b_i(t), H] = \varepsilon_i b_i(t) - (-1)^{Bn_i(t)} \sum_{j \in \partial i} t_{ij} b_j(t), \quad (4)$$

where the sum runs over the neighbors of i . We are interested in the decay of the correlation function at large distance. In analogy to the fermionic (single particle) study by Nguyen et al. [3], we restrict ourselves to forward scattering paths to leading order in t/W . Hence, we retain only the neighbors j , which are closest to 0, cf. Fig. 1. To leading order, we can also neglect the time dependence of $n_i(t)$ and approximate $(-1)^{n_i(t)} = \text{sign}(\varepsilon_i) + O((t/W)^2)$.

To characterize the spatial decay of an excitation of given energy, it is preferable to work in frequency space, $G_{i,0}^R(\omega) = \int_{-\infty}^{\infty} G_{i,0}^R(t) e^{i\omega t} dt$, and to *define* the boson localization length as the (log-averaged) inverse decay rate of $G_{i,0}^R(\omega)$ with distance,

$$\xi(\omega)^{-1} = - \lim_{\vec{r}_i \rightarrow \infty} \overline{\ln[|G_{i,0}^R(\omega)/G_{0,0}^R(\omega)|]/|\vec{r}_i - \vec{r}_0|}. \quad (5)$$

The frequency dependence will be discussed further below. Note that the transition to the superfluid is indicated by the divergence of $\xi(\omega = 0)$, where the bosons condense into a delocalized state forming at the chemical potential.

To leading order in t the above equations furnish a simple recursion relation for the Green's functions with increasing distance. Upon iteration, this forward scattering approximation (FSA) yields $G_{i,0}^R$ as a sum over all shortest paths \mathcal{P} (of length ℓ) between sites 0 and i ,

$$\frac{G_{i,0}^R(\omega)}{G_{0,0}^R(\omega)} = \sum_{\mathcal{P}=\{j_0=0, \dots, j_\ell=i\}} \prod_{p=1}^{\ell} \frac{t_{j_{p-1}, j_p} [\text{sgn}(\varepsilon_{j_p})]^B}{\varepsilon_{j_p} - \omega}. \quad (6)$$

Setting $\omega = \varepsilon_0$ we find the "wavefunction" of the quasiparticle excitation, which is adiabatically connected to the boson insertion/removal at site 0 in the non-hopping limit (by extracting the residue of the corresponding pole in $G_{i,0}^R$). The forward scattering and the particle rearrangements on a given path are illustrated in Fig. 1.

For fermions, Eq. (6) reproduces the result for non-interacting particles [2, 3], while hard core bosons differ crucially in the sign of the amplitude contributed by the paths. The result is conveniently rewritten as

$$\frac{G_{i,0}^{R,\text{bos}}(\omega)}{G_{0,0}^{R,\text{bos}}(\omega)} = \sum_{\mathcal{P}=\{j_0=0, \dots, j_\ell=i\}} \prod_{p=1}^{\ell} \frac{t_{j_{p-1}, j_p}}{|\varepsilon_{j_p}| - \text{sgn}(\varepsilon_{j_p})\omega} \quad (7)$$

The difference is simple to understand. In order to observe a particle at site i , in response to inserting a particle at 0, all the $n_P \equiv \sum_{j=1}^{\ell} n_j \approx \sum_{j=1}^{\ell} (1 - \text{sgn}(\varepsilon_j))/2$ particles on the path \mathcal{P} have to move to the next negative energy site closer to site i , cf. Fig. 1. Upon removing the particle at site i , a ring exchange of n_P particles has been carried out in the ground state, which causes the sign difference $(-1)^{n_P}$ between bosonic and fermionic amplitudes. This difference has important consequences and will shed new light not only on strongly localized insulators, but also on the approach to superfluidity.

Most importantly, Eq. (7) shows that for low energy excitations, $\omega \rightarrow 0$, in the absence of a magnetic field, all paths contribute a positive amplitude and therefore interfere constructively, unlike fermions. This difference manifests itself in completely opposite response to a magnetic field. It is well known that hopping fermions experience a negative MR due to the suppression of destructive interference [3, 5]. In contrast, we find the MR of bosons to be strongly positive, as the phases in the hopping amplitudes reduce the constructive interference of paths connecting the sites relevant for transport. The replica scaling arguments of Ref. [4], which maps the forward scattering problem to directed polymers, should apply also to the bosonic case. Here it predicts a *negative* perturbative correction to the hopping probability scaling as $Br_{\text{hop}}^{3/2}$, where r_{hop} is the hopping distance. At larger magnetic field, one finds a reduction of the boson localization length, whose effect on the MR is exponentially amplified in the hopping regime.

The opposite interference behavior of bosons and fermions is very likely a key element for understanding the giant MR peak in disordered films with remnant pairing. As long as the magnetic field does not destroy the localized Cooper pairs, it mainly reduces the localization length of the pairs. Upon destruction of the pairs, e.g., by the Zeeman effect, the predominant carriers are fermions, for which a negative MR due to increased localization length is predicted [3, 4]. Once the latter becomes large, the physics of loops and weak localization (neglected in FSA) is likely to play a role in the negative MR, as well. The effects of Coulomb interactions enhance the negative MR even further [32].

Let us now discuss how bosonic insulators differ from fermionic ones. A beautiful series of experiments on periodically patterned Bi films has shown [11] that Aharonov-Bohm oscillations in the MR start with an up- rather than a down-turn of resistance at low fields. This was interpreted as a signature of bosonic carriers, for which we provide theoretical support here. Further, the enhancement of forward scattering due to constructive interference implies that hard core bosons have a larger localization length than fermions, when subject to the same hopping and disorder. Moreover, Eq. (7) predicts that $\xi(\omega)$ has a non-trivial energy dependence around $\omega = 0$, reaching a maximum at the chemical potential $\omega = 0$, as

is easily confirmed numerically. In fact, the presence of other bosons enhances the delocalization tendency of an extra particle, whereas non-interacting fermions are insensitive to the presence of the Fermi sea. On the other hand, at high energies bosons tend to behave like non-interacting particles, as paths through occupied sites become negligible.

We point out, that the discussion of $\xi(\omega)$ is based on the FSA, which yields a recursive relation between Green's functions at the *same* ω , like in a non-interacting problem. However, this "elastic" behavior breaks down when subleading loops are taken into account and the interacting nature of the problem becomes manifest. Insofar, the frequency dependence of $\xi(\omega)$ makes sense a priori on intermediate length scales, where the FSA dominates the propagation amplitude. Instead, at large scales, $\xi(\omega)$, defined by Eq. (5), must always be an increasing function of ω . This phenomenology is closely analogous to that of phonons. Their localization length increases with decreasing energy, which is well-defined as long as inelastic scattering can be neglected.

The reduced localization tendency in low energy FSA is in conflict with predictions of Refs. [13, 24], where the presence of hard core bosons was argued to impede the propagation of an injected boson. While the arguments of [13] apply to a *distinguishable* extra particle, they neglect the exchange effects of identical particles, which in fact enhance the propagation. Similar ideas may have inspired the Bethe lattice study in [24]. Their perturbation series in t for $(G_{i0}^R(\omega))^2$ was restricted to the subset of intermediate one particle excited states, yielding path amplitudes as a product of $1/(|\varepsilon_i| - \omega)$ instead of expression Eq. (7). Since this subtle difference changes the results qualitatively, we use (7) here to revisit the analysis of Ref. [24] for hard core bosons on a Bethe lattice of large connectivity K . The latter ensures that the FSA is well-controlled up to very close to the superfluid. On a Bethe lattice, only a single shortest path connects two sites, interference phenomena being subleading in t , and statistics is irrelevant, since no particles are exchanged. Indeed, one finds the same localization properties for hard core bosons as for fermions [33], as characterized by $(G_{i0}^R(\omega))^2$ at large distances.

Neglecting loop corrections to the FSA, the superfluid transition occurs when $\sum_{i, \text{dist}(i,0)=R \rightarrow \infty} G_{i0}^R(\omega = 0)$ diverges, which happens at the same $(t/W)_c = O(1/K \ln(K))$ as the delocalization of single particles [33]. This is ensured by the fact that the sums $\sum_{i, \text{dist}(i,0)=R \rightarrow \infty} (G_{i0}^R(\omega = 0))^\alpha$ ($\alpha = 1, 2$) are both dominated by the same rare paths, which can be regarded as replica symmetry breaking [2, 24]. In the insulator we find results which deviate from [24]: we find that $G_{i0}^R(\omega)$ never decays more slowly than $G_{i0}^R(0)$, suggesting that excitations at higher energy are also localized. Since at large K collisions of single boson excitations are very inefficient in enhancing delocalizations, as argued in [24],

the above might imply that the insulator is fully localized even at extensive energies. We believe, however, that this abrupt jump from superfluid to "superinsulator" is an artifact of the large K limit.

This and other features are indeed expected to be modified at finite connectivity K , and certainly in finite dimensions, where the interference of multiple paths is crucial. However, one aspect of the FSA analysis at large K , is expected to have more general validity. The above suggested that the superfluid transition occurs as a delocalization phenomenon at $\omega = 0$, *without* concomitant closing of a mobility gap. Since effects beyond FSA seem to reinforce this feature, we conjecture it to be quite general at disorder driven SI transitions. This contrasts with the criticality of the mobility edge proposed in [13, 24] and similar early ideas by Hertz et al. [34], while being consistent with the conjectures of Ref. [23]. We emphasize, however, that the Bose glass adjacent to the superfluid is in general not fully localized at all energies of order $O(1)$. Instead, it may possess a finite, but non-critical mobility gap, which can give rise to activated transport as discussed in [13]. An exactly solvable model with such properties will be presented elsewhere [36].

The bosonic locator expansion may help to understand how bosons can escape localization in two dimensions, while fermions generically localize in the absence of spin orbit interactions. The usual argument proceeds by showing that a superfluid is stable to weak disorder [35]. Our approach complements this view from the insulating side: At low energies, all paths interfere constructively (which is a precursor of the establishment of a global phase in the superfluid). Hence, the special role played by time reversed paths for fermions seems negligible for bosons, since most paths interfere constructively anyway.

In conclusion, we have shown that strongly disordered bosons exhibit localization properties and response to a magnetic field which is completely opposite to that of fermions. It would be interesting to look for signatures of the predicted localization features, as reflected by the dependence $\xi(\omega, B)$. Apart from superconducting films, promising experimental systems are cold bosonic atoms [28], where a magnetic field can be simulated by a rotating frame. We hope that the locator expansion may be pushed to approach critical phenomena and fractality at the superfluid-insulator transition.

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